

On invariant quantization of non-Abelian gauge fields

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A strong coupling expansion around the non-trivial extremum of the Yang-Mills action will be described. It is shown that the developed formalism is the Gribov ambiguity free and each order of the developed perturbation theory is transparently gauge invariant. The result is a consequence of the restriction: calculations are not going beyond the module square of the S -matrix elements.

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1. INTRODUCTION

We present a partial solution of the Gribov problem¹⁻³: it is impossible to extract unambiguously the non-Abelian gauge symmetry degrees of freedom by Faddeev-Popov *ansatz* if the gauge field is strong, see also^{4,5}. At the same time the canonical quantization scheme certainly prescribes to extract the symmetry degrees of freedom⁶. Therefore, the problem of quantization of the non-Abelian gauge theories is on hand.

Our aim is to show that the problem can be solved if the module square of the amplitude, $|\mathbb{A}|^2$, is calculated. The quantization problem becomes simpler since the phase of \mathbb{A} is excluded from consideration in this case. In other words, we will argue that the Yang-Mills quantum field theory is free from Gribov ambiguities if it is used for the phase-free quantities description. The application of this restricted formalism was deduced in a number of papers, see e.g.^{7,8}.

It can be shown⁹ that functional integral representation for $|\mathbb{A}|^2$ is defined on the δ -like (Dirac) measure:

$$DM(A) = \prod_{x,a} dA_{a\mu}(x) \delta \left(\frac{\delta S(A)}{\delta A^{a\mu}(x)} + \hbar J_{a\mu}(x) \right), \quad (1)$$

where $A_{a\mu}$ is the Yang-Mills potential, a is the color index. It is important that no gauge fixing procedure is assumed for derivation of (1), see Sec.2. The Dirac measures appearance is the consequence of cancelations: the phase of \mathbb{A} can stay arbitrary when the measurables, $\sim \mathbb{A}\mathbb{A}^\dagger$, are calculated, Sec.2.

We will consider in present paper the solution in the frame of which quantum source $\hbar J_{a\mu}(x)$ is switched on adiabatically, i.e. we will searching for a solution of the equation:

$$\frac{\delta S(A)}{\delta A^{a\mu}(x)} + \hbar J_{a\mu}(x) = 0 \quad (2)$$

in the form of power series over $\hbar J_{a\mu}(x)$. The "generalized correspondence principle" written in (1) is strict for arbitrary value of \hbar and, therefore, the functional integral defined on the measure (1) permits the arbitrary transformations^a). The point is that (1) defines the rule how the quantum force, $J_{a\mu}(x)$, must be transformed if the field, $A_{a\mu}(x)$, is transformed.

It is the Dirac measure requires to perform the transformation in the class of strict solutions, $u_{a\mu}(x)$, of the sourceless (with $J_{a\mu} = 0$) Lagrange equation. This stands for mapping into the coset space W ,¹⁰⁻¹⁴:

$$u_{a\mu} : A_{a\mu} \rightarrow \{\lambda\} \in W. \quad (3)$$

Here W corresponds to the factor group \mathcal{G}/\mathcal{H} , where \mathcal{G} is the symmetry group of the problem and \mathcal{H} is the invariance group of $u_{a\mu}$. The qualitative reason of this choice is following: after having got the ground state field $u_{a\mu}(x)$, where $u_{a\mu}(x)$ is any strict solution of sourceless Lagrange equation, the freedom in the choice of the value of integration constants, $\{\lambda\}$, is what remains from the continuum of the field degrees of freedom, see Sec.3. The gauge phases $\{\Lambda^a\}$ must be included in $\{\lambda\}$.

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It should be noted that the mapping (3) may be singular if $\dim W$ is finite. We will show that the singularity can be isolated (and canceled by the normalization). This is our renormalization procedure. Sec. 4 contains description of this procedure.

It will be shown that each order of our perturbation series is transparently gauge invariant since the gauge-invariant, $\mathbb{A}\mathbb{A}^\dagger$, will be calculated. This will be shown in Sec. 5. Therefore, no gauge fixing procedure is required and no ambiguities appears. This is the main result. The preliminary version of the formalism was given in¹⁵.

2. PERTURBATION THEORY

We will consider the theory with the action:

$$S(A) = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a(A) F_a^{\mu\nu}(A) \quad (4)$$

The Yang-Mills fields

$$F_{a\mu\nu}(A) = \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} - f_{abc} A_{b\mu} A_{c\nu} \quad (5)$$

are the non-Abelian gauge covariants. The group will not be specified. The matrix notation: $A_{a\mu}\omega_a = A_\mu$ will be also used.

We will calculate the quantity^{b)}:

$$\mathcal{N} = |\mathcal{Z}|^2, \quad (6)$$

where the vacuum-into-vacuum transition amplitude

$$\mathcal{Z} = \int DA e^{iS_C(A)}, \quad DA = \prod_{\mathbf{x}, t \in C} \prod_{a, \mu} \frac{dA_{a, \mu}(\mathbf{x}, t)}{\sqrt{2\pi}}, \quad (7)$$

is defined on the Minkowski metric. The Mills complex time formalism will be used to avoid the possible light cone singularities¹⁶. For example, the theory may be defined on the complex time contour

$$C: t \rightarrow t + i\varepsilon, \quad \varepsilon \rightarrow +0, \quad -\infty \leq t \leq +\infty. \quad (8)$$

At the very end one must take $\varepsilon = +0$. The Mills formalism restores the Feynman's $i\varepsilon$ -prescription.

2.1. Dirac measure

The double integral:

$$\mathcal{N} = \int DA^+ DA^- e^{iS_C(A^+) - iS_C^*(A^-)} \quad (9)$$

will be calculated. To extract the Dirac measure^{c)}, one can introduce the mean trajectory, $A_{a\mu}$, and the virtual deviation, $a_{a\mu}$, instead of A_μ^\pm :

$$A_{a\mu}^\pm(x) = A_{a\mu}(x) \pm a_{a\mu}(x). \quad (10)$$

The transformation (10) is linear and the differential measure

$$DA^+ DA^- = \prod_{\mathbf{x}, t \in C+C^*} \prod_{a, \mu} dA_{a\mu}(x) \prod_{\mathbf{x}, t \in C+C^*} \prod_{a, \mu} \frac{da_{a\mu}(x)}{\pi} \equiv DADa. \quad (11)$$

is defined on the entire time contour $C + C^*$.

The "closed-path" boundary conditions:

$$a_{a\mu}(x \in \sigma_\infty) = 0, \quad (12)$$

where σ_∞ is the remote time-like hypersurface, is assumed. We will demand that the surface terms are cancelled in the difference $S_C(A^+) - S_C^*(A^-)$, i.e.

$$\int dx \partial_\mu (A_{a\nu} \partial^\mu A^{a\nu})^+ = \int dx \partial_\mu (A_{a\nu} \partial^\mu A^{a\nu})^- \quad (13)$$

if (12) is taken into account. Therefore, not only the trivial pure gauge fields can be considered on σ_∞ .

Expanding $S(A \pm a)$ over $a_{a\mu}$, one can write:

$$S_C(A + a) - S_C^*(A - a) = U(A, a) + 2\text{Re} \int_C dx a_{a\mu}(x) \frac{\delta S(A)}{\delta A_{a\mu}(x)}. \quad (14)$$

This equality will be used as the definition of the remaining term, $U(A, a)$. With the ε accuracy, $U(A, a) = O(a^3)$, i.e. $U(A, a)$ introduces the interactions.

Noticing that

$$\frac{\delta S(A)}{\delta A^{a\mu}(x)} = D_a^{\nu b} F_{\mu\nu}^b, \quad (15)$$

where $D_a^{\nu b}$ is the covariant derivative and inserting (11) and (14) into (9), we find:

$$\mathcal{N} = \int DA \int Da e^{2i\text{Re} \int_C dx a_a^\mu(x) D_a^{\nu b} F_{\mu\nu}^b} e^{iU(A, a)}. \quad (16)$$

The integrals over $a_{a\mu}(x)$ will be calculated perturbatively. For this purpose one can use the identity:

$$e^{iU(A, a)} = \lim_{\zeta_{a\mu}=J_{a\mu}=0} e^{-i\mathbb{K}(J, \zeta)} e^{2i\text{Re} \int_C dx a_{a\mu}(x) J^{a\mu}(x)} e^{iU(A, \zeta)}, \quad (17)$$

where

$$2\mathbb{K}(J, \zeta) = \text{Re} \int_C dx \frac{\delta}{\delta J^{a\mu}(x)} \frac{\delta}{\delta \zeta_{a\mu}(x)}. \quad (18)$$

In the future we will omit the symbol of the limit appearing in (17) keeping in mind the prescription: the auxiliary variables, $J_{a\mu}$ and $\zeta_{a\mu}$, must be taken equal to zero at the very end of calculations.

Assuming that the perturbation series will exist, the insertion of the Eq.(17) into (16) gives the desired expression:

$$\mathcal{N} = e^{-i\mathbb{K}(J, \zeta)} \int DM(A) e^{iU(A, \zeta)}, \quad (19)$$

where

$$\begin{aligned} DM(A) &= \prod_{\mathbf{x}, t \in C+C^*} \prod_{a, \mu} dA_{a\mu}(x) \int \prod_{\mathbf{x}, t \in C+C^*} \prod_{a, \mu} \frac{da_{a\mu}(x)}{\pi} e^{2i\text{Re} \int_C dx a_a^\mu (D_a^{\nu b} F_{\mu\nu}^b - J_{a\mu}(x))} = \\ &= \prod_{\mathbf{x}, t \in C+C^*} \prod_{a, \mu} dA_{a\mu}(x) \delta(D_a^{\nu b} F_{\mu\nu}^b - J_{a\mu}(x)) \end{aligned} \quad (20)$$

is the functional Dirac measure. The functional δ -function on the complex time contour $C + C^*$ has the definition:

$$\prod_{\mathbf{x}, t \in C+C^*} \delta(D_a^{\nu b} F_{\mu\nu}^b - J_{a\mu}) = \prod_{\mathbf{x}, t \in C} \delta(\text{Re}(D_a^{\nu b} F_{\mu\nu}^b - J_{a\mu})) \delta(i\text{Im}(D_a^{\nu b} F_{\mu\nu}^b - J_{a\mu})). \quad (21)$$

It is important that the exponent in (20) is pure imaginari as the consequence of the fact that the module $|\mathcal{Z}|^2$ is calculated.

It can be shown that (19) gives the ordinary perturbation theory (pQCD)¹⁷⁻²⁰ if the equation:

$$D_{ab}^\nu F_{\mu\nu}^b = J_{a\mu} \quad (22)$$

is expanded in the vicinity of $A_{a\mu} = 0$. Notice that the Eq.(22) is not gauge invariant because of $J_{a\mu}(x)$.

3. MAPPING INTO THE COSET SPACE

We will formulate the general method of mapping (3) into the infinite dimensional phase space Γ_∞ , Sec.3.2, and then will find the reduction procedure, $\Gamma_\infty \mapsto W$ on the second stage of the calculation, Sec.4.1.

3.1. First order formalism

The action in terms of the electric field, $P_a^i = F_a^{i0}$, $i = 1, 2, 3$, looks as follows:

$$S(\mathbf{A}, \mathbf{E}) = \frac{1}{g^2} \int dx \left\{ \mathbf{P}_a \dot{\mathbf{A}}_a - \frac{1}{2} (\mathbf{P}_a^2 + \mathbf{B}_a^2) + A_{0a} (\mathcal{D}\mathbf{P})_a \right\}, \quad (23)$$

where the magnetic field $\mathbf{B}_a(\mathbf{A}) = \text{rot} \mathbf{A}_a - \frac{1}{2} (\mathbf{A} \times \mathbf{A})_a$ and $(\mathcal{D}\mathbf{P})_a = \partial_i P_{ai} - f_{abc} A_{bi} P_{ci}$. The corresponding Dirac measure is:

$$DM(\mathbf{A}, \mathbf{P}) = \prod_a \prod_x d\mathbf{A}_a(x) d\mathbf{P}_a(x) \delta(\mathcal{D}\mathbf{P}_a) \delta \left(\dot{\mathbf{P}}_a(x) + \frac{\delta H_J(\mathbf{A}, \mathbf{P})}{\delta \mathbf{A}_a(x)} \right) \delta \left(\dot{\mathbf{A}}_a(x) - \frac{\delta H_J(\mathbf{A}, \mathbf{P})}{\delta \mathbf{P}_a(x)} \right), \quad (24)$$

where $d\mathbf{A}_a(x) d\mathbf{P}_a(x) = \prod_i dA_{ia}(x) dP_{ai}(x)$, $i = 1, 2, 3$, and the total Hamiltonian

$$H_J = \frac{1}{2} \int d^3x (\mathbf{P}_a^2 + \mathbf{B}_a^2) - \int d^3x \mathbf{J}_a \mathbf{A}_a. \quad (25)$$

Notice that the dependence on A_{a0} was integrated out and as a result the Gauss law, $\mathcal{D}_b^a \mathbf{P}_b = 0$, was appeared in (24). The Faddeev-Popov *ansatz* was not used for the definition of the integral over $A_{a\mu}$. The perturbations generating operator \mathbb{K} and the remainder potential term U stay unchanged, see (18) and (14).

The integrals with the measure (24) will be calculated using new "collective-like" variables. The same was proposed by Faddeev and authors⁴. But they introduce the condition $\mathcal{D}_b^a \mathbf{P}_b = 0$ by hands and their transformation to the new variables leads to the complicated singular Hamiltonian.

3.2. General mechanism of transformations

Proposition 1. *The Jacobian of transformation (3) of the Dirac measure (24) is equal to one¹⁷. One can insert the unite*

$$1 = \frac{1}{\Delta(\lambda, \kappa)} \int \prod_{\alpha, t} d\lambda_\alpha(t) d\kappa_\alpha(t) \prod_{a, \mathbf{x}} \delta(\mathbf{A}_a(\mathbf{x}, t) - \mathbf{u}_a(\mathbf{x}; \lambda, \kappa)) \delta(\mathbf{P}_a(\mathbf{x}, t) - \mathbf{p}_a(\mathbf{x}; \lambda, \kappa)) \quad (26)$$

into the integral (19) and integrate over \mathbf{A}_a and \mathbf{P}_a using the δ -functions of (26). This is one way to perform the transformation. Otherwise, if the δ -functions of (24) are used, \mathbf{u}_a and \mathbf{p}_a will play the role of constraints. It must be noted that the both ways of calculation must lead to the identical ultimate result because of the δ -likeness of measures in (24) and (26). The first way is preferable since it does not imply the ambiguous gauge fixing procedure¹⁻³.

To be correct the power of sets (λ, κ) and (\mathbf{A}, \mathbf{P}) must coincide since in this case only one may introduce transformations like (3). For this purpose we will consider the theory on the *space* lattice.

The given composite functions $\mathbf{u}_a(\mathbf{x}; \lambda(t), \kappa(t))$ and $\mathbf{p}_a(\mathbf{x}; \lambda(t), \kappa(t))$ must obey the condition:

$$\Delta(\lambda, \kappa) = \int \prod_{\alpha, t} d\lambda'_\alpha(t) d\kappa'_\alpha(t) \prod_a \delta(\lambda' \mathbf{u}_{a, \lambda} + \kappa' \mathbf{u}_{a, \kappa}) \delta(\lambda' \mathbf{p}_{a, \lambda} + \kappa' \mathbf{p}_{a, \kappa}) \neq 0, \quad (27)$$

where

$$\mathbf{u}_{a, X} \equiv \frac{\partial \mathbf{u}_a}{\partial X}, \quad \mathbf{p}_{a, X} \equiv \frac{\partial \mathbf{p}_a}{\partial X}, \quad X = (\lambda_\alpha, \kappa_\alpha)(t)$$

and (λ, κ) are the solutions of the equations:

$$\mathbf{A}_a(\mathbf{x}, t) - \mathbf{u}_a(\mathbf{x}; \lambda, \kappa) = 0, \quad \mathbf{P}_a(\mathbf{x}, t) - \mathbf{p}_a(\mathbf{x}; \lambda, \kappa) = 0. \quad (28)$$

The summation over the repeated index, α , will be assumed. It must be underlined that the functions (\mathbf{u}, \mathbf{p}) are given. Therefore, the equalities (28) restrict the form of functions (\mathbf{A}, \mathbf{P}) on the measure (24).

The transformed measure:

$$DM(\lambda, \kappa) = \frac{1}{\Delta(\lambda, \kappa)} \prod_{\alpha, t} d\lambda_\alpha(t) d\kappa_\alpha(t) \prod_a \delta \left(\dot{\lambda}_{\mathbf{u}_a, \lambda} + \dot{\kappa}_{\mathbf{u}_a, \kappa} - \frac{\delta H_J(\mathbf{u}, \mathbf{p})}{\delta \mathbf{p}_a} \right) d \left(\dot{\lambda}_{\mathbf{p}_a, \lambda} + \dot{\kappa}_{\mathbf{p}_a, \kappa} + \frac{\delta H_J(\mathbf{u}, \mathbf{p})}{\delta \mathbf{u}_a} \right) \quad (29)$$

can be diagonalize introducing the auxiliary function(al) h_J :

$$\begin{aligned} DM(\lambda, \kappa) = & \frac{1}{\Delta(\lambda, \kappa)} \prod_{\alpha, t} d\lambda_\alpha(t) d\kappa_\alpha(t) \int \prod_{\alpha, t} d\lambda'_\alpha(t) d\kappa'_\alpha(t) \\ & \times \delta \left(\lambda'_\alpha - \left(\dot{\lambda}_\alpha - \frac{\delta h_J(\lambda, \kappa)}{\delta \kappa_\alpha} \right) \right) \delta \left(\kappa'_\alpha - \left(\dot{\kappa}_\alpha + \frac{\delta h_J(\lambda, \kappa)}{\delta \lambda_\alpha} \right) \right) \\ & \times \prod_a \delta \left(\mathbf{u}_{a, \lambda} \lambda' + \mathbf{u}_{a, \kappa} \kappa' + \{\mathbf{u}, h_J\}_a - \frac{\delta H_J}{\delta \mathbf{p}_a} \right) \delta \left(\mathbf{p}_{a, \lambda} \lambda' + \mathbf{p}_{a, \kappa} \kappa' + \{\mathbf{p}, h_J\}_a + \frac{\delta H_J}{\delta \mathbf{u}_a} \right), \end{aligned} \quad (30)$$

where $\{, \}$ is the Poisson bracket and (λ, κ) are the solution of equations

$$\dot{\lambda}_{\mathbf{u}_a, \lambda} + \dot{\kappa}_{\mathbf{u}_a, \kappa} - \frac{\delta H_J(\mathbf{u}, \mathbf{p})}{\delta \mathbf{p}_a} = 0, \quad \dot{\lambda}_{\mathbf{p}_a, \lambda} + \dot{\kappa}_{\mathbf{p}_a, \kappa} + \frac{\delta H_J(\mathbf{u}, \mathbf{p})}{\delta \mathbf{u}_a} = 0, \quad (31)$$

see (29).

Let us assume now that \mathbf{u}_a , \mathbf{p}_a and h_J are chosen in such a way that:

$$\{\mathbf{u}_a, h_J\} - \frac{\delta H_J}{\delta \mathbf{p}_a} = 0, \quad \{\mathbf{p}_a, h_J\} + \frac{\delta H_J}{\delta \mathbf{u}_a} = 0. \quad (32)$$

Then, having the condition (27), the transformed measure takes the form, see (30):

$$DM(\lambda, \kappa) = \prod_{\alpha, t} d\lambda_\alpha(\mathbf{x}, t) d\kappa_\alpha(\mathbf{x}, t) \delta \left(\dot{\lambda}_\alpha(\mathbf{x}, t) - \frac{\delta h_J(\lambda, \kappa)}{\delta \kappa_\alpha(\mathbf{x}, t)} \right) \delta \left(\dot{\kappa}_\alpha(\mathbf{x}, t) + \frac{\delta h_J(\lambda, \kappa)}{\delta \lambda_\alpha(\mathbf{x}, t)} \right), \quad (33)$$

where the functional determinant $\Delta(\lambda, \kappa)$ was canceled since the sets (λ, κ) in (30) and (27) must coincide if

$$h_J(\lambda, \kappa) = H_J(\mathbf{u}, \mathbf{p}), \quad (34)$$

i.e. if h_J is the transformed "Hamiltonian". One can find the prove in **Proposition 3**.

As a result,

$$\mathcal{N} = e^{-i\mathbb{K}(J, \zeta)} \int DM(\lambda, \kappa) e^{iU(\mathbf{u}, \zeta)} \quad (35)$$

where $\mathbb{K}(J, \zeta)$ was defined in (18), $DM(\lambda, \kappa)$ was defined in (33) and $U(\mathbf{u}, \zeta)$ was introduced in (14). Therefore, the Jacobian of transformation is equal to one, i.e. in the frame of the conditions (27) and (34) the phase space volume is conserved. Q.E.D.

According to (25) the transformed hamiltonian h_J is:

$$h_J(\lambda, \kappa) = h(\lambda, \kappa) - \int d\mathbf{x} \mathbf{J}_a(\mathbf{x}, t) \mathbf{u}_a(\mathbf{x}; \lambda, \kappa). \quad (36)$$

Therefore, we come to the following dynamical problem:

$$\dot{\lambda}_\alpha = \frac{\delta h_J(\lambda, \kappa)}{\delta \kappa_\alpha} = \frac{\delta h(\lambda, \kappa)}{\delta \kappa_\alpha} - \int d\mathbf{x} \mathbf{J}_a \frac{\delta \mathbf{u}_a}{\delta \kappa_\alpha} \equiv h_{\kappa_\alpha} - \int d\mathbf{x} \mathbf{J}_a \mathbf{u}_{a, \kappa_\alpha}, \quad (37)$$

$$\dot{\kappa}_\alpha = -\frac{\delta h_J(\lambda, \kappa)}{\delta \lambda_\alpha} = -\frac{\delta h(\lambda, \kappa)}{\delta \lambda_\alpha} + \int d\mathbf{x} \mathbf{J}_a \frac{\delta \mathbf{u}_a}{\delta \lambda_\alpha} \equiv -h_{\lambda_\alpha} + \int d\mathbf{x} \mathbf{J}_a \mathbf{u}_{a, \lambda_\alpha}. \quad (38)$$

Proposition 2. *If (36) is held and the perturbation series exists then the transformation (3) induces the splitting:*

$$\mathbf{J}_a \rightarrow \{j_\lambda, j_\kappa\} \quad (39)$$

The proof of the splitting comes from the identity:

$$\prod_{\alpha,t} \delta \left(\dot{\lambda}_\alpha - \frac{\delta h_J(\lambda, \kappa)}{\delta \kappa_\alpha} \right) \delta \left(\dot{\kappa}_\alpha + \frac{\delta h_J(\lambda, \kappa)}{\delta \lambda_\alpha} \right) = \exp\{-i\mathbf{k}(j, e)\} \exp \left\{ 2i \text{Re} \int_C d\mathbf{x} dt \mathbf{J}_a(\mathbf{x}, t) (e_{\kappa_\alpha} \mathbf{u}_{a, \lambda_\alpha} - e_{\lambda_\alpha} \mathbf{u}_{a, \kappa_\alpha}) \right\} \\ \times \prod_{\alpha,t} \delta(\dot{\lambda}_\alpha - h_{\kappa_\alpha} - j_{\lambda_\alpha}) \delta(\dot{\kappa}_\alpha + h_{\lambda_\alpha} - j_{\kappa_\alpha}), \quad (40)$$

where

$$2\mathbf{k}(j, e) = \text{Re} \int_C dt dx \left(\frac{\delta}{\delta j_{\lambda_\alpha}} \frac{\delta}{\delta e_{\lambda_\alpha}} + \frac{\delta}{\delta j_{\kappa_\alpha}} \frac{\delta}{\delta e_{\kappa_\alpha}} \right). \quad (41)$$

At the very end one must take $j_X = e_X = 0$, $X = (\lambda, \kappa)$. The equality (40) can be derived using the functional δ -functions Fourier transformation (20).

Inserting (40) into (35) and using linearity over \mathbf{J}_a of the exponent in (40), we find the completely transformed representation for \mathcal{N} , where the individual to each degree of freedom quantum sources, j_X , $X = (\lambda, \kappa)$, appears. The transformed representation of \mathcal{N} looks like:

$$\mathcal{N} = e^{-i\mathbf{k}(j, e)} \int DM(\lambda, \kappa) e^{iU(\mathbf{u}, \mathbf{e})}, \quad (42)$$

where

$$DM(\lambda, \kappa) = \prod_{\alpha, \mathbf{x}, t} d\lambda_\alpha(\mathbf{x}, t) d\kappa_\alpha(\mathbf{x}, t) \delta(\dot{\lambda} - h_\kappa(\lambda, k) - j_\lambda) \delta(\dot{\kappa} + h_\lambda(\lambda, k) - j_\kappa), \quad (43)$$

$$\mathbf{e}_a = e_{\kappa_\alpha} \mathbf{u}_{a, \lambda_\alpha} - e_{\lambda_\alpha} \mathbf{u}_{a, \kappa_\alpha} \quad (44)$$

and $\mathbf{k}(j, e)$ was defined in (41). Q.E.D.

Proposition 3. *The Eqs. (32) and the measure (43) define the classical flow. Indeed,*

$$\dot{\mathbf{u}}_a = \dot{\lambda} \mathbf{u}_{a, \lambda} + \dot{\kappa} \mathbf{u}_{a, \kappa} = \{\mathbf{u}_a, h_J\} = \frac{\delta H_J}{\delta \mathbf{p}_a}, \\ \dot{\mathbf{p}}_a = \dot{\lambda} \mathbf{p}_{a, \lambda} + \dot{\kappa} \mathbf{p}_{a, \kappa} = \{\mathbf{p}_a, h_J\} = -\frac{\delta H_J}{\delta \mathbf{u}_a}, \quad (45)$$

where (43) and then (32) have been used step by step. Therefore, \mathbf{u}_a is the solution of the sourceless Lagrange equation (22) and $\mathbf{p}_a = \dot{\mathbf{u}}_a$. Q.E.D.

Proposition 3 means that α in (26) is the coset space index and the condition (27) is satisfied.

3.3. An example of coset space: scalar theory

Let us start from conformal φ^4 theory. The exact $O(4) \times O(2)$ invariant solution for this theory is known^{21,22}:

$$u(\mathbf{x}, t) = \left\{ \frac{-(\gamma - \gamma^*)^2}{(x - \gamma)^2 (x - \gamma^*)^2} \right\}^{1/2}. \quad (46)$$

The "Hamiltonian" looks as follows:

$$H(u, p) = \int d\mathbf{x} \left(\frac{1}{2} p^2 - \frac{1}{2} (\partial_i u)^2 + \frac{1}{4} g u^4 \right). \quad (47)$$

The time-like complex vector

$$\gamma_\mu = \xi_\mu + i\eta_\mu, \quad \gamma_\mu \gamma^\mu = \gamma_0^2 - \gamma_i^2, \quad i = 1, 2, 3 \quad (48)$$

and ξ_μ and η_μ are the real numbers. The parameters $\{\xi, \eta\}$ form the coset space W . Their physical domain is defined by inequalities:

$$-\infty \leq \xi_\mu \leq +\infty, \quad -\infty \leq \eta_i \leq +\infty, \quad \eta_\mu \eta^\mu \geq 0. \quad (49)$$

The solution (46) is regular in the Minkowski metric for $\eta = \sqrt{\eta_\mu \eta^\mu} \geq 0$ and has the pole singularity at

$$(x - \xi)^2 = 0 \quad (50)$$

if $\eta = 0$. We will regularize it continuing on the Mills complex-time contour¹⁶. The solution (46) has the finite energy and no topological charge. There also exist its elliptic generalizations of (46)²².

Let us consider now (ξ, η) as the dynamical variables:

$$(\xi, \eta) = (\xi, \eta)(\mathbf{x}, t) \quad (51)$$

assuming that the solution of equations:

$$\{u, h_J\} - \frac{\delta H_J}{\delta p} = 0, \quad \{p, h_J\} + \frac{\delta H_J}{\delta u} = 0,$$

see (32) and

$$\dot{\xi}(\mathbf{x}, t) - \frac{\delta h_J(\xi, \eta)}{\delta \eta(\mathbf{x}, t)} = 0, \quad \dot{\eta}_\alpha(\mathbf{x}, t) + \frac{\delta h_J(\xi, \eta)}{\delta \xi(\mathbf{x}, t)} = 0,$$

see (33), where

$$h_J(\xi, \eta) = H_J(u, p),$$

coincides with (46) in the classical limit $J = 0$. In that limit we have the equations:

$$\begin{aligned} \sum_\alpha \left(\frac{\partial u}{\partial \xi_\alpha(\mathbf{x}, t)} \frac{\delta h}{\delta \eta_\alpha(\mathbf{x}, t)} - \frac{\partial u}{\partial \eta_\alpha(\mathbf{x}, t)} \frac{\delta h}{\delta \xi_\alpha(\mathbf{x}, t)} \right) &= \frac{\delta H}{\delta p(\mathbf{x}; \xi, \eta)} = p(\mathbf{x}; \xi, \eta), \\ \sum_\alpha \left(\frac{\partial p}{\partial \xi_\alpha(\mathbf{x}, t)} \frac{\delta h}{\delta \eta_\alpha(\mathbf{x}, t)} - \frac{\partial p}{\partial \eta_\alpha(\mathbf{x}, t)} \frac{\delta h}{\delta \xi_\alpha(\mathbf{x}, t)} \right) &= -\frac{\delta H}{\delta u(\mathbf{x}; \xi, \eta)} = -(\partial_i^2 u + gu^3)(\mathbf{x}; \xi, \eta) \end{aligned} \quad (52)$$

and

$$\dot{\xi}_\alpha(\mathbf{x}, t) - \frac{\delta h(\xi, \eta)}{\delta \eta_\alpha(\mathbf{x}, t)} = 0, \quad \dot{\eta}_\alpha(\mathbf{x}, t) + \frac{\delta h(\xi, \eta)}{\delta \xi_\alpha(\mathbf{x}, t)} = 0. \quad (53)$$

for $u(\mathbf{x}; \xi, \eta)$ and $p(\mathbf{x}; \xi, \eta)$. First equation in (52) means that

$$\{u(\mathbf{x}; \xi, \eta), u(\mathbf{x}; \xi, \eta)\} \frac{\delta H}{\delta u(\mathbf{x}, t)} + (\{u(\mathbf{x}; \xi, \eta), p(\mathbf{x}; \xi, \eta)\} - 1) \frac{\delta H}{\delta p(\mathbf{x}, t)} = 0.$$

Therefore, u and p must obey the equation:

$$\sum_\alpha \left(\frac{\partial u(\mathbf{x}; \xi, \eta)}{\partial \xi_\alpha(\mathbf{x}, t)} \frac{\delta p(\mathbf{x}; \xi, \eta)}{\delta \eta_\alpha(\mathbf{x}, t)} - \frac{\partial u(\mathbf{x}; \xi, \eta)}{\partial \eta_\alpha(\mathbf{x}, t)} \frac{\delta p(\mathbf{x}; \xi, \eta)}{\delta \xi_\alpha(\mathbf{x}, t)} \right) = \{u(\mathbf{x}; \xi, \eta), p(\mathbf{x}; \xi, \eta)\} = 1. \quad (54)$$

But this is impossible solution since u and p are not the canonically conjugated coordinate and momentum in the considered theory with symmetry. It must be noted that (54) is the unique consequence of Eqs. (52).

One can consider another variables:

$$\xi = \xi(t), \quad \eta = \eta(t). \quad (55)$$

In this case one can find:

$$\int d\mathbf{y} \{u(\mathbf{x}; \xi, \eta), u(\mathbf{y}; \xi, \eta)\} \frac{\delta H}{\delta u(\mathbf{y}, t)} + \int d\mathbf{y} (\{u(\mathbf{x}; \xi, \eta), p(\mathbf{y}; \xi, \eta)\} - \delta(\mathbf{x} - \mathbf{y})) \frac{\delta H}{\delta p(\mathbf{y}, t)} = 0. \quad (56)$$

One may consider following solution of this equation:

$$\{u(\mathbf{x}; \xi, \eta), u(\mathbf{y}; \xi, \eta)\} = 0, \quad \{u(\mathbf{x}; \xi, \eta), p(\mathbf{y}; \xi, \eta)\} = \delta(\mathbf{x} - \mathbf{y})$$

which is equivalent of (54) and must be rejected if $\delta H/\delta u(\mathbf{y}, t)$ and $\delta H/\delta p(\mathbf{y}, t)$ are not the independent quantities. Therefore Eq. (56) must be satisfied only in the integral over the 3-coordinate sense. Just that case is realised.

Let us assume now that the variables η_α and ξ_α are chosen so that

$$h = h(\eta). \quad (57)$$

In this case Eqs. (53) looks as follows:

$$\dot{\xi}_\alpha(t) = \frac{\delta h(\eta)}{\delta \eta_\alpha(t)} \equiv \omega_\alpha(\eta), \quad \dot{\eta}_\alpha(t) = -\frac{\delta h(\eta)}{\delta \xi_\alpha(t)} = 0 \quad (58)$$

i.e. in the considered semi-classical approximation η_α are the integrals of motion and $\delta h(\eta)/\delta \eta_\alpha(t) = \omega_\alpha(\eta)$ are constant velocities in the factor space W , i.e.

$$\xi_\alpha(t) = \omega_\alpha(\eta^0)(t - t^0), \quad \eta_\alpha(t) = \eta_\alpha^0, \quad (59)$$

where t^0 and η_α^0 are the time independent constants.

3.4. An example: Non-Abelian gauge theory

The Gribov ambiguity actually presents the problem in the non-Abelian gauge theory since we know, at least, the $O(4) \times O(2)$ -invariant strict solution of the $SU(2)$ Yang-Mills equation. The corresponding coset space has $\dim W = 8$ plus the (infinite) gauge groups dimension.

The *ansatz*^{23,24}:

$$\sqrt{g}u_\mu^a = \eta_{\mu\nu}^a \partial^\nu \ln u \quad (60)$$

for the $SU(2)$ Yang-Mills potential u_μ^a leads to the conformal scalar, φ^4 , field theory, see e.g.²², which was considered in previous subsection.

4. REDUCTION

After having done the mapping, see (42)-(44), one must extract from the set $\{\kappa, \lambda\}$ the dynamical variables $\{\xi, \eta\}$.

4.1. Cyclic variables

Let us divide the set $\{\lambda, \kappa\}$ into two parts:

$$\{\lambda, \kappa\} \rightarrow (\{\lambda, \kappa\}, \{\xi', \eta'\}), \quad (61)$$

assuming that λ and κ are cyclic variables:

$$\frac{\partial \mathbf{u}_a}{\partial \lambda} \approx 0, \quad \frac{\partial \mathbf{u}_a}{\partial \kappa} \approx 0 \quad (62)$$

and the derivatives of \mathbf{u}_a over ξ' and η' not vanish at $\varepsilon = 0$. It can be shown that the variables (λ, κ) stay cyclic in the quantum sense as well.

Proposition 4. *The quantum force is orthogonal to the cyclic variables axes.*

Indeed, taking into account (62),

$$\mathbf{k}(j, e) = \int dt \left\{ \frac{\delta}{\delta j_\lambda} \cdot \frac{\delta}{\delta e_\lambda} + \frac{\delta}{\delta j_\kappa} \cdot \frac{\delta}{\delta e_\kappa} + \frac{\delta}{\delta j_{\xi'}} \cdot \frac{\delta}{\delta e_{\xi'}} + \frac{\delta}{\delta j_{\eta'}} \cdot \frac{\delta}{\delta e_{\eta'}} \right\}. \quad (63)$$

As it follows from (43),

$$\frac{\delta \mathbf{u}_a}{\delta j_X} \sim \frac{\delta \mathbf{u}_a}{\delta X} \approx 0, \quad X = (\lambda, \kappa). \quad (64)$$

Therefore, we can write taking into account (62) and (64) that

$$2\mathbf{k}(j, e) = \int dt \left\{ \frac{\delta}{\delta j_{\xi'}} \cdot \frac{\delta}{\delta e_{\xi'}} + \frac{\delta}{\delta j_{\eta'}} \cdot \frac{\delta}{\delta e_{\eta'}} \right\}. \quad (65)$$

Then, following our definition, one should take everywhere

$$j_X = e_X = 0, \quad X = (\lambda, \kappa). \quad (66)$$

The result of the reduction looks as follows:

$$DM(u, p) = d\Omega \, DM(\xi', \eta'), \quad (67)$$

where the infinite dimensional integral over

$$d\Omega = \prod_{\alpha, t} d\lambda_\alpha(t) d\kappa_\alpha(t) \delta(\dot{\lambda}_\alpha(t)) \delta(\dot{\kappa}_\alpha(t)) \quad (68)$$

will be cancelled by normalization. This procedure completes the renormalization in the transformed formalism.

The remaining degrees of freedom are entered into the reduced Dirac measure:

$$DM(\xi', \eta') = \prod_t d\xi'(t) d\eta'(t) \delta(\dot{\xi}' - h_{\eta'}(\xi', \eta') - j_{\xi'}) \delta(\dot{\eta}' + h_{\xi'}(\xi', \eta') - j_{\eta'}). \quad (69)$$

This result presents the first step of the reduction into the physical coset space W . Q.E.D.

Let us consider now the case when only the part of variables are cyclic: $\{\xi'\} = (\{\xi\}, \{\xi''\})$ and $\{\eta'\} = (\{\eta\}, \{\eta''\})$,

$$\dim\{\xi\} = \dim\{\eta\}$$

where, for example, only $\{\xi''\}$ is the set of cyclic variables (the case when $\{\eta''\}$ is cyclic is similar):

$$\frac{\partial \mathbf{u}_a}{\partial \xi''} \approx 0. \quad (70)$$

It can be easily show that we come to the condition of Proposition 4, and in this case the conjugated variables $\{\eta''\}$ are the integrals of motion.

Indeed, in the frame of the definition (70) we have

$$DM(\xi, \eta; \eta'') = \prod_t d\xi(t) d\eta(t) d\xi''(t) d\eta''(t) \delta(\dot{\xi} - h_\eta - j_\xi) \delta(\dot{\eta} + h_\xi - j_\eta) \delta(\dot{\eta}'' - j_{\eta''}). \quad (71)$$

Following (44) the virtual deviation \mathbf{e} looks as follows:

$$\mathbf{e}_a = e_\eta \mathbf{u}_{a\xi} - e_\xi \mathbf{u}_{a\eta} + e_{\eta''} \mathbf{u}_{a\xi''} \quad (72)$$

and the perturbations generating operator is:

$$2\mathbf{k}(j, e) = \int dt \left\{ \frac{\delta}{\delta j_\xi} \frac{\delta}{\delta e_\xi} + \frac{\delta}{\delta j_\eta} \frac{\delta}{\delta e_\eta} + \frac{\delta}{\delta j_{\eta''}} \frac{\delta}{\delta e_{\eta''}} \right\}. \quad (73)$$

As it follows from the general condition that the auxiliary variables must be taken equal to zero, we must put $e_{\eta''} = 0$ because of (70), (72). We must omit simultaneously the last term in (73). For this reason one must put $j_{\eta''} = 0$ in (71) and therefore η''_α are the integrals of motion.

Following to this section one can conclude that gauge degrees of freedom can not belong to the quantum variables, $\{\Lambda_a\} \not\subset \{\xi, \eta\}$, since there is no conjugated to Λ^a gauge charge dependence in the field $u_{a\mu}$.

4.2. Concluding expression

As a result,

$$\mathcal{N} = e^{-i\mathbf{k}(je)} \int DM(\xi, \eta) e^{iU(\mathbf{u}, \mathbf{e})}, \quad (74)$$

where the new coset space virtual deviation is

$$\mathbf{e}_a = \sum_{\alpha} \left\{ e_{\eta_{\alpha}} \frac{\partial \mathbf{u}_a}{\partial \xi_{\alpha}} - e_{\xi_{\alpha}} \frac{\partial \mathbf{u}_a}{\partial \eta_{\alpha}} \right\}. \quad (75)$$

The generating quantum perturbations operator in the coset space is

$$2\mathbf{k}(je) = \sum_{\alpha} \int dt \left\{ \frac{\delta}{\delta j_{\xi_{\alpha}}(t)} \frac{\delta}{\delta e_{\xi_{\alpha}}(t)} + \frac{\delta}{\delta j_{\eta_{\alpha}}(t)} \frac{\delta}{\delta e_{\eta_{\alpha}}(t)} \right\}, \quad (76)$$

where summation is performed over all canonical pairs, $(\xi, \eta) \in T^*W$. Let us choose the variables $\{\xi, \eta\}$ so that $\partial h / \partial \xi = 0$, then the corresponding measure is

$$DM = dR \prod_{\alpha, t} d\xi_{\alpha}(t) d\eta_{\alpha}(t) \delta(\dot{\xi}_{\alpha} - h_{\eta_{\alpha}}(\eta) - j_{\xi_{\alpha}}) \delta(\dot{\eta}_{\alpha} - j_{\eta_{\alpha}}), \quad (77)$$

where dR is the zero modes Cauchy measure:

$$dR = \prod_{\alpha, t} d\eta_{\alpha}'' \delta(\dot{\eta}_{\alpha}''). \quad (78)$$

Therefore,

$$W = T^*W + R \quad (79)$$

where $\{\xi, \eta\} \in T^*W$ and $\{\xi''\} \in R$.

The coset space Hamiltonian equations:

$$\dot{\xi}_{\alpha} - h_{\eta_{\alpha}}(\eta) = j_{\xi_{\alpha}}, \quad \dot{\eta}_{\alpha} = j_{\eta_{\alpha}} \quad (80)$$

are easily solved through the Green function $g(t - t')$. The latter must obey the equation:

$$\partial_t g(t - t') = \delta(t - t'). \quad (81)$$

This Green function has the universal meaning, and it must be the same for arbitrary theory. Then, using the $i\varepsilon$ -prescription and the experience of the Coulomb problem considered in¹⁷, we will use the following solution of (81):

$$g(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}. \quad (82)$$

The solution of the Eq.(80) looks as follows:

$$\xi_a^j(t) = \int dt' g(t - t') \{h_{\eta_{\alpha}}(\eta^j) + j_{\xi_{\alpha}}\}(t'), \quad (83)$$

$$\eta_a^j(t) = \int dt g(t - t') j_{\eta_{\alpha}}(t'). \quad (84)$$

As a result, the functional measure DM is reduced to the Cauchy measure

$$dM = \prod_{\alpha, t} d\eta_{\alpha}''(t) \delta(\dot{\eta}_{\alpha}'') d\xi_{\alpha}(t) d\eta_{\alpha}(t) \delta(\dot{\xi}_{\alpha}) \delta(\dot{\eta}_{\alpha}) = \prod_{\alpha} d\eta_{\alpha}''(0) d\xi_{\alpha}(0) d\eta_{\alpha}(0). \quad (85)$$

The integral over dM gives the volume V of the factor group \mathcal{G}/\mathcal{H} and $\dim V \leq \dim W$.

Notice that the gauge group volume V_{Λ} in our formalism is defined by the measure $\prod_{a, \mathbf{x}} d\Lambda_a(\mathbf{x}, 0)$.

Therefore,

$$\mathcal{N} = e^{-i\mathbf{k}(je)} \int dM e^{iU(\mathbf{u}^j, \mathbf{e}^j)}, \quad (86)$$

where \mathbf{u}^j and \mathbf{e}^j depends on the functions (ξ_a^j, η_a^j) .

5. GAUGE INVARIANCE

The new coset space virtual deviation

$$\mathbf{e}_a = \sum_{\alpha} \left\{ e_{\eta_{\alpha}} \frac{\partial \mathbf{u}_a}{\partial \xi_{\alpha}} - e_{\xi_{\alpha}} \frac{\partial \mathbf{u}_a}{\partial \eta_{\alpha}} \right\}. \quad (87)$$

is the covariant of gauge transformations: if

$$u_{a\mu} \rightarrow \Omega u_{a\mu} \Omega^{-1} + i \Omega \partial_{\mu} \Omega^{-1} \quad (88)$$

then

$$\mathbf{e}_a \rightarrow \Omega \mathbf{e}_a \Omega^{-1}, \quad (89)$$

because of the condition $\{\Lambda_a\} \not\subseteq \{\xi, \eta\}$.

Following (14) and (75),

$$\begin{aligned} U(u, e) &= S(u + e) - S^*(u - e) - 2\text{Re} \sum_{a, \alpha} \int_C dx \left\{ e_{\eta_{\alpha}} \frac{\partial u_{ai}(x)}{\partial \xi_{\alpha}} - e_{\xi_{\alpha}} \frac{\partial u_{ai}(x)}{\partial \eta_{\alpha}} \right\} \frac{\delta S(u)}{\delta u_{ai}(x)} \\ &= S(u + e) - S^*(u - e) - 2\text{Re} \sum_{\alpha} \int_C dt \left\{ e_{\eta_{\alpha}}(t) \frac{\delta}{\delta \xi_{\alpha}(t)} - e_{\xi_{\alpha}}(t) \frac{\delta}{\delta \eta_{\alpha}(t)} \right\} S(u). \end{aligned} \quad (90)$$

This quantity is transparently gauge invariant, since the action S is the invariant of gauge transformation (88), (89).

We can conclude that each term of the coset space perturbation theory is gauge invariant since DM in (77) and $\mathbf{k}(je)$ in (76) are the gauge invariant quantities.

It is interesting to note that in spite of the fact that each term of the perturbation theory is transparently gauge invariant, nevertheless, one can not formulate the theory only in terms of the gauge field strength.

6. CONCLUSIONS

It is useful to summarize the rules of the coset space perturbation theory.

(i) The transformation, (3), to independent variables is performed having in mind that the power of the variables set must not be altered.

(ii) The "host free" transformation is induced by the function \mathbf{u}_a defined by the Eq.(32). In this stage the function $h_j(\lambda, \kappa)$ is arbitrary.

(iii) If h_J is the transformed Hamiltonian, $h_J = h + \int dx \mathbf{u}_a \mathbf{J}_a$, then there exists a mapping into the W space, see (3). This mapping produces a new set of sources $\{j_{\lambda}, j_{\kappa}\}$, (41), and virtual deviations, $\{e_{\lambda}, e_{\kappa}\}$, (44). It is remarkable that each degree of freedom of the (λ, κ) space is excited independently of one another by the individual sources $\{j_{\lambda}, j_{\kappa}\}$. This is crucial for the reduction of the quantum degrees of freedom.

(iv) One can consider the case when a subset of variables is cyclic, see (61) and (62). As a result we have found the reduced measure (69), and the perturbations generating operator (65). The volume of the cyclic variables, (68), is cancelled by normalization. The field theoretical problem becomes finite dimensional. The cancellation of the cyclic variables volume can be considered as a renormalization procedure.

(v) W is the coset space. The choice of the coset variables $\{\xi, \eta\}$ is arbitrary.

(vi) A portion of the remaining variables can belong to the symplectic subspace $T^*W \subseteq W$. The latter allows to conclude that the gauge phase Λ_a can not belong to T^*W . As a result the perturbation theory is transparently gauge invariant.

(vii) The known solution²¹ shows that all space-time integrals of the coset space perturbation theory are finite outside the border ∂W since $|S(u)| < \infty$ and $\dim W$ is finite. The border contributions, $\sup(\xi, \eta) \in \partial W$, remain finite because of the $i\varepsilon$ -prescription. Further analysis of the role of the border singularities, see also¹⁷, will be given in subsequent publications.

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Notes

^{a)}Notice that the general transformations of functional integral leads to the wrong results^{25,26}, see also⁵.

^{b)}The generalization was considered in^{8,27}.

^{c)}The term " δ -like (Dirac) measure" have been taken from²⁸.

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